Tight Sets, m-ovoids, and Strongly Regular Cayley Graphs

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Based on joint work with Tao Feng, Weicong Li, Koji Momihara, Morgan Rodgers, and Hanlin Zou.

Abstract: In this series of talks, we will discuss a few constructions of tight sets and m-ovoids in finite classical polar spaces. Special emphasis will be on the recent construction of Cameron-Liebler line classes in $\mathrm{PG}(3,q)$.

Cameron-Liebler line classes are sets of lines in $\mathrm{PG}(3,q)$ having many interesting combinatorial properties. These line classes were first introduced by Cameron and Liebler in their study of collineation groups of $\mathrm{PG}(3,q)$ having the same number of orbits on points and lines of $\mathrm{PG}(3,q)$. During the past decade, Cameron-Liebler line classes have received considerable attention from researchers in both finite geometry and algebraic combinatorics. In the original paper [1] by Cameron and Liebler, the authors gave several equivalent conditions for a set of lines of $\mathrm{PG}(3,q)$ to be a Cameron-Liebler line class; later Penttila gave a few more of such characterizations. We will use one of these characterizations as the definition of Cameron-Liebler line class. Let $\mathcal{L}$ be a set of lines of $\mathrm{PG}(3,q)$ with $|L| = x(q^2+q+1)$, x a positive integer. We say that $\mathcal{L}$ is a Cameron-Liebler line class with parameter x if every spread of $\mathrm{PG}(3,q)$ contains x lines of $\mathcal{L}$. It turned out that Cameron-Liebler line classes are closely related to certain subsets of points (tight sets) of the Klein quadric. We will talk about a recent construction in [2] of a new infinite family of Cameron-Liebler line classes with parameter $x = (q+1)^2/3$ for $q \equiv 2 \pmod 3$. When q is an odd power of 2, this family of Cameron-Liebler line classes represents the first infinite family of Cameron-Liebler line classes ever constructed in $\mathrm{PG}(3,q)$, q even. This talk is based on joint work with Tao Feng, Koji Momihara, Morgan Rodgers and Hanlin Zou.

References

[1] P. J. Cameron, R. A. Liebler, Tactical decompositions and orbits of projective groups, Linear Algebra Appl. 46 (1982), 91–102.

[2] T. Feng, K. Momihara, M. Rodgers, Q. Xiang, H. Zou, Cameron-Liebler line classes with parameter $x = \frac{(q+1)^2}{3}$, Advances in Math. 385 (2021), 107780.

[3] T. Feng, Weicong Li, Q. Xiang, A new family of (q^4+1)-tight sets with an automorphism group $F_4(q)$, Elec. J. Combin. 32 (2025), no.2, Paper No.2.25